PORTUGALIAE MATHEMATICA

VOLUME 34

1975

Edição de

«GAZETA DE MATEMÁTICA, LDA»

PORTUGALIAE MATHEMATICA
Rua Diário de Notícias, 134, 1.º-Esq.
LISBOA-2 (PORTUGAL)

PORTUGALIAE MATHEMATICA
Vol. 34 — Fasc. 4 — 1975

A DIRECT SUM REPRESENTATION (1)

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Norms defined by supremums are in general not generated by scalar products. However, using a representation theorem of BOCHNER; HARDY spaces ($p = 2$) on tube domains are represented as HILBERT spaces.

Let T be a tube in C^n with base S, i.e. $T = \{Z \mid Z \in C^n, R(Z) \in S\}$, $S = \{(x_1, \cdots, x_n) \mid \sigma_i < x_i < \eta_i\}$. $H^2(T) = \{f \mid f$ holomorphic in T,

$$\|f\|^2 = \sup_{x \in S} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |f(x + iy)|^2 \, dy_1 \, dy_2 \cdots dy_n < \infty\}.$$

THEOREM. $H^2(T)$ *is isomorphic and homeomorphic to the direct sum of* 2^n *copies of* $H^2(T_0)$ *where the base of* T_0 *is given by* $\sigma_1 = \sigma_2 = \cdots = \sigma_n = 0$, $\eta_1 = \eta_2 = \cdots = \eta_n = +\infty$.

PROOF. Because of the invariance of the integral, under affine transformations, used in computing the norm, it is sufficient to show that f has a unique representation of the form $f = f_1 + \cdots + f_{2^n}$ where $f_j \in H^2(T_j)$, T_j an octant-shaped tube. T_j is octant shaped if for each i, either $\sigma_i = -\infty$ or $\eta_i = +\infty$ but not both. We must also show that the mapping $f \rightarrow (f_1, \cdots f_{2^n})$ is 1:1 and bi-continuous. It is 1:1 and onto if the representation is unique.

The desired unique representation is provided by BOCHNER's Theorem [1]. It then remains to establish continuity. Clearly $\|f\| \leq \|f_1\| + \cdots + \|f_{2^n}\|$ where $\|f_j\|$ is the norm of f_j in $H^2(T_j)$,

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(1) Received April, 1974.
AMS 1970 subject classifications 32-A07, 30-A78, 46-E20, 46-J15. Key phrases: Tube Domains, Hilbert spaces functions, isomorphisms holomorphic.

hence by the Open Mapping Theorem the inverse map $(f_1, \cdots, f_{2^n}) \to f$ is also continuous.

Since $H^2(T_0)$ is a HILBERT space, $H^2(T)$ is isomorphic and homeomorphic to the HILBERT space $\oplus \sum_{j=1}^{2^n} H^2(T_0)$. Using this mapping we may also imbed $H^2(T)$ as a finite dimensional subspace of the square summable power series on the unit disk with coefficients in $H^2(T_0)$.

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